# Chapter 2. Interest and cash flow 



GV: Lê Thị Thanh An

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### 2.1. Indexes



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### 2.1. Indexes

### 2.1.1. Percentage change - Sự thay đổi phần trăm

A percentage is a number expressed as a fraction of 100 , using the sign "\%". Percentage change, often used in finance, represents the degree of change in percentage. Following we have some examples.

Example 1 (Annual sale). A firm's annual sale rises from 100 million units to 110 million units from a year to the next. Calculate the percentage change of the sale?

Example 2 (Investment value). During a year, an investment falls by 5\%. Compute the value of this investment at the end of the year, knowing that at the beginning it was approximately $\$ 145000$ ?

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2.1. Indexes

### 2.1.2. Scale factors - Hệ số tỷ lệ

In our chapter, scale factor is the ratio between a value to the base in the same unit.
Trong chương này, hệ số tỷ lệ là tỷ lệ giữa một giá trị với cơ sở theo cùng một đơn vị.
Example 3 (Commodity price). This week the price of a good was 20000 Vietnamese Dong, next week it rises to 22000 Vietnamese Dong. What is the scale factor?

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### 2.1. Indexes

Definition 1. Index is a number used to show the value of something by comparing it to something else whose value is known:

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\text { Index = scale factor } \times 100 .
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Example 4 (Share comparison). Compare the two shares regarding returns and risks:

| Month | May | Jun | Jul | Aug | Sep | Oct |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Share A | 0.32 | 0.28 | 0.34 | 0.41 | 0.31 | 0.46 |
| Share B | 6.40 | 6.46 | 6.53 | 6.41 | 6.47 | 6.64 |

### 2.1. Indexes

### 2.1.4. Inflation - Lạm phát

Inflation is the rate that a currency purchasing power is falling or the price level is rising. Inflation could be calculated as the percentage increase in the level of prices over a 12-month period. To estimate the price level for an economy, the Consumer Price Index (CPI) and the Wholesale Price Index (WPI) are commonly used.

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### 2.1. Indexes

Solution. The average house prices in six years.

| Year | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Average house price | 70 | 85 | 88 | 95 | 102 | 105 |
| Annual rate of inflation |  | $10 \%$ | $7 \%$ | $3.6 \%$ | $2.4 \%$ | $2 \%$ |

Choose year 1 as the base year, the real data is following.

| Year | 1 | 2 | 3 | 4 | 5 | 6 |
| ---: | :--- | :--- | :---: | :---: | :---: | :---: |
| Real price | 70 | 77.27 | 74.77 | 77.91 | 81.69 | 82.44 |

So the real prices rise not much as nominal prices over six years. In Year 3, price even decreases.

### 2.2. Interest



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### 2.2. Interest

2.2.1. Interest - Lãi suất

Simple interest is calculated using the original principal amount of the loan.
Compound interest is calculated using not only the principal amount but also all the accumulated interests. In the following example we will demonstrate how to compute them.

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### 2.2. Interest

- If the interest $r$ is compounded $n$ time per year, then in $t$ years we have $n t$ periods. The interest for each period is $r / n$, so the scale factor is $(1+r / n)$. A formula to compute the future value $F$ of present value $P$ is

$$
F=P\left(1+\frac{r}{n}\right)^{n t} .
$$

- If the interest $r$ is compounded continously, then

$$
F=P \lim _{n \rightarrow+\infty}\left(1+\frac{r}{n}\right)^{n t}=P e^{r t} .
$$

### 2.2. Interest

Example 6 (Simple and compound interests). Suppose we invest \$1000 for two years. Compute the final sum in case of
a. simple interest, which is $8 \%$ annually.
b. compound interest, which is $8 \%$ compounded annually.
c. compound interest, which is $8 \%$ compounded semiannually.

### 2.2. Interest

2.2.2. Annual percentage rate - Lãi suất phần trăm hàng năm

Annual equivalent rate or annual percentage rate (APR) is the annual interest needed for a loan, after taking into account all accumulated interests according to the contract.

$$
A P R=\left(1+\frac{r}{n}\right)^{n}-1 .
$$

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### 2.2. Interest and discounting

Example 7 (Annual equivalent rate). Suppose Maria want to open a saving account. In the followings, which one is the better options:
a. The nominal interest is $8.2 \%$ compounded annually.
b. The nominal interest is $8 \%$ compounded semi-annually.

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### 2.2. Interest

### 2.2.3. Discounting - Chiết khấu

- If the interest $r$ is compounded $n$ times per year, the future value $F$ in $t$ years is equivalent to the present value P :

$$
P=F\left(1+\frac{r}{n}\right)^{-n t}
$$

- If n goes to infinity, then the interest $r$ is the discount rate compounded continuously, and we get:

$$
P=F e^{-r t} .
$$

### 2.2. Interest

Example 8 (Discounting). The predicted apartment price is $\$ 100000$ in 4 years. How much is the corresponding price today if the discount rate is $6 \%$
a. compounded annually
b. compounded continuously.

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### 2.2. Interest

### 2.2.4. Annuity - Niên kim

An annuity is a contract, often used by insurance companies, where their customers first make a lump-sum payment or series of payments and later can receive some regular disbursements.

Example 9 (Annuity). Suppose that for the next 20 years, a customer wants to receive a regular income of $\$ 12000$ at the end of each year. The discount rate for this cash flow is $6 \%$, compounded every year. Compute the present sum?

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### 2.3. Investment appraisal

### 2.3.1. Net present value (NPV) - giá trị hiện tại ròng

The net present value (NPV) of an investment is equal to the present value of the revenue flow minus the present sum of all related costs. The project is worthwhile if the NPV is positive. For the same cost, which project has the higher NPV is better.

Example 10 (Net present value). You are considering to invest $\$ 7500$ in a project. In four years, you are certain that it brings out $\$ 10000$. The market rate for discount is $5 \%$, compounded annually. Calculate the net present value and comment on the project?

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### 2.3. Investment appraisal

### 2.3.2. Internal rate of return (IRR) - Tỷ suất hoàn vốn nội bộ

The internal rate of return (IRR) is the annual interest rate for which the NPV equals to zero. When using IRR for the same cost in the same time period, we obtain the same return.

Example 11 (Internal rate of return). Find the IRR for project in example above, knowing that it requires $\$ 7500$ initial cost and produces $\$ 10000$ in four years? Given the market rate is $5 \%$, comment on this project?

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### 2.3. Investment appraisal

### 2.3.3. Investment appraisal - Thẩm định đầu tư

If there are several different projects, we would prefer the one provides the highest positive net present value. However, depending on the given information, the internal rate of return may be more convenient to use, especially in case there is only one investment.

Example 12 (Housing investment). Investing in an apartment in 2021 in a big city would require 2 billion VND. We assume the price in two years is 2.1 billion VND. A person would like to rent the apartment in two years with 144 million VND, the amount is paid in 2021, at the beginning of the contract.
a. The current market rate is $6 \%$. Comment on this investment?
b. Comment on the investment if the market rate is $7 \%$ ?

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### 2.3. Investment appraisal

Example 13 (Profit comparison). There are two projects, both require initial investment of $\$ 12000$. In return, each project provides a flow of income as in Table 7. Which project is more attractive, if the market rate is $10 \%$ compounded annually?

Table 7. Revenue streams for two projects.

|  | Revenue (\$) |  |
| :--- | :--- | :--- |
| In the end of the year | Project A | Project B |
| 1 | 3000 | 1500 |
| 2 | 3000 | 1500 |
| 3 | 4500 | 3000 |
| 4 | 4500 | 7000 |
| 5 | 4500 | 7500 |

## THANK YOU

 02837244555 www.uel.edu.vn-- ĐẠI HỌC QUỐC GIA THÀNH PHỐ HỔ CHÍ MINH TRUỜNG DÅl HOC KINH TẾ - LUÂT
Só́ 669 Quốc lộ 1, Khu phó 3, Phuòng Linh Xuân, TP. Thư Đúc, TP. Hồ Chí Minh.

